.CONSTRUCTING CONFIDENCE INTERVALS USING THE BOOTSTRAP: AN APPLICATION TO A MULTI-PRODUCT COST FUNCTION

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Abstract—A multi-product cost function system is estimated for 387 banks in states that allow branch banking. The bootstrap resampling method is used to construct confidence intervals for marginal costs, output-cost elasticities, economies of scale and scope, and Allen elasticities of substitution. Confidence intervals for these measures are usually constructed using a first-order variance approximation under a normality assumption, but such confidence intervals are inexact if the measures are not normally distributed or the variance approximations are imprecise. We find that the bootstrap standard error estimates can differ significantly from the usual estimates. Furthermore, we use the bootstrap to expand the analysis of cost function regularity properties.

I. Introduction

Often researchers are interested in statistics that are nonlinear combinations of an empirical model's estimated parameters, which are generally assumed to be normally distributed. For a cost function these measures include marginal cost or scale economies, partial elasticities of substitution, and, in the multiple output case, measures of scope economies. Nonlinearity makes standard confidence intervals unreliable because a nonlinear function of normally distributed parameter estimates will not necessarily follow a normal or other convenient distribution. Thus, the usefulness of traditional standard error estimates is questionable. Also, regularity conditions, which cannot easily be imposed (such as concavity of a cost function), are typically verified at the means of the data or possibly at each data point. However, formal tests that the conditions truly hold are currently unavailable.

Confidence intervals for measures that are functions of parameter estimates can be easily constructed using a bootstrap resampling technique. Perhaps more importantly, bootstrap resampling also allows one to assign a statement of confidence to the conclusion that regularity conditions are satisfied. In this paper, we construct alternative empirical confidence intervals for commonly used cost function measures, and compare them to intervals calculated using approximate standard errors. Bootstrap resampling also allows us to state that regularity conditions are likely satisfied in this model.

Bootstrap resampling provides an empirical substitute for the often impossible alternative of deriving the theoretical distribution of a complex statistic. The bootstrap has a relatively high start-up cost because of the Monte Carlo type estimations. However, because the tedium of constructing gradient vectors is avoided, there is a very low incremental cost as the number of statistics for which confidence intervals are desired increases. Consequently, if the number of statistics is great enough, bootstrap resampling is a low cost substitute for the traditional confidence interval even if the underlying distributions are known.

We present four alternative methods of constructing confidence intervals and compare these alternatives to the traditional approach. Our findings are valid only for the data set we use and one should be cautious in attempting to generalize. However, in this paper we do make three generalizations. First, bootstrap resampling is useful in evaluating the validity of t-statistics for testing hypotheses about nonlinear combinations of parameter estimates. Second, in many cases bootstrap resampling provides a low cost method of obtaining confidence intervals. Third, bootstrap resampling allows the researcher to make statements of confidence about properties such as regularity which heretofore have been extremely difficult to verify statistically.

II. Bootstrap Confidence Intervals

Bootstrap methods are well-suited to constructing standard error estimates and confidence intervals when the sample size is small or the distribution of the statistic is unknown. Although sample size is not a problem for our data set, it is a common problem in studies that employ "flexible form" functions, such as the translog function, because of the many parameters estimated. The distribution can be a problem because many statistics calculated from an estimated cost function are nonlinear functions of variables that are commonly assumed to be normally distributed.

First-order approximations to the standard errors can be calculated using an approximate variance formula (Kmenta, 1986, pp. 486–487). These estimates may be imprecise if higher-order terms in the Taylor expansion are important. Also, the usefulness of accurate standard error estimates is unclear if the underlying distributions are unknown. Often authors assume

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normal distributions, but there is, in general, no reason to expect complex statistics to follow a normal distribution. Furthermore, there is evidence that this is incorrect for Allen elasticities of substitution (Anderson and Thursby, 1986). Constructing confidence intervals for such measures is precisely the sort of problem for which bootstrap methods are appropriate.

Using a Monte Carlo algorithm, we estimate the model B times. We first estimate our model of m equations using all n data points. We call these results our base model estimates, and let v denote an estimate of V, the statistic of interest. We keep the $n \times m$ matrix of errors from the base model estimation and the predicted values of the dependent variables. We then randomly draw with replacement n times from the rows of the error matrix, form a new $n \times m$ error matrix, and construct new dependent variables by adding the new error matrix to the $n \times m$ matrix of predicted values. The probability that the new matrix of dependent variables is identical to the original one is n^{-n}.

We re-estimate the model B times, using B different dependent variable matrices. This yields B estimates of V, denoted $v^*(b)$ where $b = 1, \ldots, B$. The bootstrap estimate of the standard error (s_B) is the standard deviation of $v^*(b)$, the bootstrap estimates. That is, s_B is the square root of $\Sigma_{b=1}^{B}[v^*(b) - v^*(.)]^2/(B - 1)$, where $v^*(.) = \Sigma_{b=1}^{B} v^*(b)/B$. As B increases, s_B approaches the population bootstrap standard error[1]. In practice, $B = 100$ appears adequate for estimates of s_B, whereas $B = 1000$ is appropriate for estimates of confidence intervals (Efron and Tibshirani, 1986, and Efron, 1987).

The *basic bootstrap confidence interval* is $c_s(\alpha) = v \pm s_B z^{(\alpha)}$ where $z^{(\alpha)}$ is the 100α percentile point of a standard normal distribution. Construction of this confidence interval is identical to the construction of traditional confidence intervals except that the approximate standard error is replaced by the bootstrap estimate of the standard error. This confidence interval is correct if v is distributed normally with constant variance. More general confidence intervals are presented in Efron and Tibshirani. The simplest of these is the *percentile method*, which is constructed by ordering v^* and choosing critical value observations as the endpoints of the confidence intervals. (For example, if $B = 1000$, observations 26 and 975 are the endpoints of the 95% confidence interval.) This interval, denoted c_p, is appropriate if there exists a monotone transformation of v such that the transformation is normally distributed with constant variance.

If the distribution is symmetric, but the estimate of V is biased, then the percentile method is inaccurate. Instead, *the bias-corrected confidence interval*, c_{BC}, is appropriate. To construct c_{BC} define $\hat{G}(v)$ as the percentage of $v^*(b)$ that are less than v, and let $z_0 = \Phi^{-1}\{\hat{G}(v)\}$, where Φ^{-1} is the inverse function of the standard normal distribution function. For a 95% confidence interval, c_{BC} is constructed by choosing the values of $v^*(b)$ that are in the $100\Phi\{2z_0 \pm 1.96\}$ percentiles. If $\hat{G}(v) = 0.5$, then $z_0 = 0$ and c_{BC} collapses to c_p. If $\hat{G}(v)$ is less (greater) than 0.5, then z_0 is less (greater) than 0 and the confidence interval will be shifted, relative to c_p, so that both the upper and lower limits are smaller (larger) than before.

The most general confidence interval we consider is the *bias-corrected percentile*, denoted BC_a. This method corrects for both bias and skewness. A skewness measure suggested by Efron and Tibshirani is $a = (1/6)\{(\Sigma_i U_i^3)/(\Sigma_i U_i^2)^{3/2}\}$ where $U_i = (n - 1(v(.) - v(i))$, n is the number of observations in the original model, $v(i)$ is the jackknife estimate of V with the ith observation deleted, and $v(.) = \Sigma_{i=1}^{n} v(i)/n$. For a 95% confidence interval, BC_a is constructed by choosing the values of $v^*(b)$ that are in the $100\Phi\{z_0 + (z_0 \pm 1.96)/(1 - a(z_0 \pm 1.96))\}$ percentiles. If $a = 0$, BC_a is equivalent to c_{BC}.

III. The Current Application

The cost function is implicitly derived via assumptions about technology, firm behavior and input market structures. The multi-product technology, given by $T(X, Y)$, is assumed to be increasing in inputs (X), decreasing in outputs (Y), and continuous. We also assume that the firm minimizes cost and that input markets are competitive. Cost, then, is a function of outputs and input prices and inherits several properties which we call regularity conditions. These regularity conditions are the theoretical link that allows us to infer technological information from cost data.

We apply the bootstrap resampling method to estimate a multi-product cost function for a 1985 cross-section of 387 banks located in states that allow branch banking.[2] We model the bank as producing three outputs (loans, investments, and transaction deposits) using three inputs (purchased funds, labor, and capital).

[1] The bootstrap standard error is $\sigma(\hat{F})$ where $\hat{F}$ is the empirical probability distribution with probability mass $1/n$ on $x_1, x_2, \ldots, x_n$. As B approaches infinity, the standard error estimate, s_B, will approach $\sigma(F)$ (Efron and Tibshirani, 1986, pp. 54–56).

[2] We use the Federal Reserve System's Functional Cost Analysis survey. A data appendix including variable definitions and measurement as well as descriptive statistics is available from the authors.

The foundation of our empirical model is the hybrid translog cost function

$$\begin{aligned}\text{Ln}(Cost) &= \alpha_0 + \Sigma_i^3 \alpha_i Y_i + .5\Sigma_i^3\Sigma_j^3 \alpha_{ij} Y_i Y_j \\ &+ \Sigma_k^3 \beta_k \ln W_k + .5\Sigma_k^3\Sigma_n^3 \beta_{kn} \ln W_k \ln W_n \\ &+ \Sigma_i^3\Sigma_k^3 \rho_{ik} Y_i \ln W_k + \delta_n N + \Sigma_i^3 \delta_{yi} Y_i N \\ &+ \Sigma_k^3 \delta_{wk} \ln W_k N + .5\delta_{nn} N^2 \end{aligned} \quad (1)$$

where the outputs, Y_1, Y_2 and Y_3, are loans, investments and transaction deposits; W_1, W_2, and W_3 are the prices of the three inputs (purchased funds, labor and capital); and N is the number of branch offices. Shephard's lemma yields the factor share equations

$$M_k = \beta_k + \Sigma_n^3 \beta_{kn} \ln W_n + \Sigma_i^3 \rho_{ik} Y_i + \delta_{wk} N \quad \text{for } k = 1,2,3. \quad (2)$$

Our hybrid translog cost function is a special case of the generalized translog multi-product cost function developed by Caves, Christensen and Tretheway (1980). Both the hybrid translog and the translog are second-order approximations to the cost dual of T around input prices (W) and outputs (Y). The translog uses the logs of Y and W as the point of approximation while the hybrid translog uses the logs of W and levels of Y. They are equally flexible in that no restrictions are imposed on first- and second-order derivatives, which allows scale and substitution measures (the shape of T) to vary as inputs and outputs change. The hybrid translog is preferred in the multiple output case because using levels of Y allows scope economies to be evaluated, while the translog function implies zero cost if any of the outputs is not produced.

Cost minimization implies that the cost function is linearly homogeneous with respect to input prices while Young's theorem implies symmetric second cross-partial derivatives. These conditions are easily imposed restrictions.[3] Other regularity conditions, such as monotonicity, concavity and non-negative marginal costs, cannot be imposed without a great loss in functional form flexibility. Instead, these conditions are checked at each data point.

Cost function measures describing the underlying technology are marginal costs (MC), output-cost elasticities (SC), overall economies of scale (SCE), economies of scope $(SCOPE)$, and the Allen partial elasticities of input substitution (AES). These measures are

$$\begin{aligned} MC_i &= \partial C/\partial Y_i = C\partial \ln C/\partial Y_i \\ &= C\{\alpha_i + \Sigma_j^3 \alpha_{ij} Y_j + \Sigma_k^3 \rho_{ik} \ln W_k + \delta_{yi} N\} \end{aligned} \quad (3)$$

$$\begin{aligned} SC_i &= \partial \ln C/\partial \ln Y_i = Y_i \partial \ln C/\partial Y_i \\ &= Y_i\{\alpha_i + \Sigma_j^3 \alpha_{ij} Y_j + \Sigma_k^3 \rho_{ik} \ln W_k + \delta_{yi} N\} \end{aligned} \quad (4)$$

$$SCE = \Sigma_i^3 SC_i \quad (5)$$

$$SCOPE = \left[\left(\Sigma_i^3 C(Y_i, 0; W)\right) - C(Y,W)\right]/C(Y,W) \quad (6)$$

$$\begin{aligned} AES_{ii} &= (CC_{ii})/(C_i C_i) \\ &= (\beta_{ii} + M_i(M_i - 1))/(M_i M_i) \end{aligned} \quad (7)$$

$$AES_{ij} = (CC_{ij})/(C_i C_j) = 1 + \beta_{ij}/(M_i M_j), \quad (8)$$

where C is the cost function, $C_i = \partial C/\partial W_i$, $C_j = \partial C/\partial W_j$, $C_{ij} = \partial^2 C/\partial W_i \partial W_j$ and M_i and M_j are the expressions for the factor cost shares given by equation (2). We report traditional standard error estimates for these measures. Elsewhere we have implicitly assumed normality in making confidence statements (Buono and Eakin, 1990). We now use bootstrap resampling to check the validity of traditional confidence intervals.

IV. Empirical Results

We estimate the base model given by (1) and (2) by the method of iterative seemingly unrelated regressions (ITSUR).[4] We then use our base model results to re-estimate the model, via ITSUR, 1000 times using the bootstrap algorithm described in section II. The base model parameter estimates and their estimated standard errors are presented in table 1 along with the means and standard deviations of the bootstrap estimates of parameters. Very similar values are observed for the base model and bootstrap estimates.

We now present confidence intervals for the measures given by equations (3) to (8), using the estimated standard errors and the four alternative bootstrap methods. Table 2 reports, for each measure, the point estimate calculated from the base model, the traditional estimate of the standard error (s_T) calculated via the approximate variance formula, and information from the bootstrap estimates that is needed to construct the alternative confidence intervals. Comparing the traditional and bootstrap standard error estimates, we observe similar values for the marginal costs (MC_is), output-cost elasticities (SC_is), and overall economies of scale (SCE). For the scope measure and the AES measures the standard error estimates are considerably different. The large differences in the AES standard error estimates likely reflect imprecision in the first-order variance approximation. Interestingly, the prob-

[3] Homogeneity restrictions are $\Sigma_k^3 \beta_k = 1$, $\Sigma_k^3 \beta_{nk} = \Sigma_n^3 \beta_{kn} = 0$ for all k, n, $\Sigma_k^3 \rho_{ik} = 0$ for all i, and $\Sigma_k^3 \delta_{wk} = 0$. β_3, β_{13}, β_{23}, β_{33}, ρ_{13}, ρ_{23}, ρ_{33} and δ_{w3} are not estimated and not reported in table 1. Symmetry restrictions are $\alpha_{ij} = \alpha_{ji}$ for all i, j, and $\beta_{kn} = \beta_{nk}$ for all k, n.

[4] Additive errors, which may be correlated across equations but not across observations, are assumed.

TABLE 1.—PARAMETER ESTIMATES OBTAINED VIA TRADITIONAL SYSTEM ESTIMATION AND FROM 1000 BOOTSTRAP SYSTEM REGRESSIONS

	Traditional ITSUR		Bootstrap ITSUR ($B = 1000$)	
	Estimate	Std. Error	Mean	Std. Dev.
$\alpha 0$	−1.454	0.034	−1.450	0.036
$\alpha 1$	0.522	0.048	0.523	0.050
$\alpha 2$	0.353	0.057	0.355	0.058
$\alpha 3$	0.163	0.046	0.164	0.049
$\alpha 11$	−0.027	0.012	−0.027	0.012
$\alpha 12$	0.0001	0.014	0.0004	0.015
$\alpha 13$	−0.040	0.013	−0.040	0.014
$\alpha 22$	−0.027	0.024	−0.027	0.025
$\alpha 23$	−0.040	0.016	−0.040	0.016
$\alpha 33$	0.045	0.027	0.044	0.028
$\beta 1$	0.745	0.005	0.746	0.005
$\beta 2$	0.193	0.003	0.192	0.003
$\beta 11$	0.145	0.017	0.144	0.017
$\beta 12$	−0.121	0.013	−0.121	0.013
$\beta 22$	0.110	0.011	0.109	0.012
$\rho 11$	0.025	0.004	0.025	0.004
$\rho 12$	−0.020	0.003	−0.021	0.003
$\rho 21$	0.011	0.004	0.011	0.004
$\rho 22$	−0.008	0.003	−0.008	0.003
$\rho 31$	−0.020	0.004	−0.020	0.003
$\rho 32$	0.014	0.003	0.014	0.002
δN	0.038	0.007	0.037	0.008
δNN	−0.001	0.0003	−0.001	0.0003
$\delta Y1$	−0.002	0.002	−0.002	0.002
$\delta Y2$	0.001	0.003	0.001	0.003
$\delta Y3$	−0.001	0.002	−0.001	0.002
$\delta W1$	−0.002	0.0005	−0.002	0.0005
$\delta W2$	0.001	0.0004	0.001	0.0004

TABLE 2.—ESTIMATED MARGINAL COSTS (MC), OUTPUT COST ELASTICITIES (SC), ECONOMIES OF SCALE (SCE), SCOPE ECONOMIES AND ALLEN PARTIAL ELASTICITIES OF INPUT SUBSTITUTION (AES)

	v	s_T	s_B	$v^*(B) < v$ (%)	a
$MC1$	0.049	0.005	0.005	45.4	−0.072
$MC2$	0.063	0.010	0.010	49.1	−0.097
$MC3$	0.052	0.018	0.020	46.6	−0.073
$SC1$	0.438	0.040	0.041	48.4	−0.056
$SC2$	0.299	0.047	0.048	49.9	−0.093
$SC3$	0.119	0.041	0.044	47.4	−0.063
SCE	0.856	0.052	0.057	48.2	−0.092
$SCOPE$	0.659	0.067	0.074	51.3	0.074
$AES11$	−0.081	0.092	0.031	51.0	−0.004
$AES22$	−1.220	0.115	0.316	51.8	0.011
$AES33$	−11.790	0.332	0.629	50.4	0.003
$AES12$	0.154	0.030	0.094	49.5	−0.002
$AES13$	0.491	0.307	0.115	48.9	0.006
$AES23$	1.923	0.641	0.336	51.5	−0.004

TABLE 3.—CONFIDENCE INTERVALS: TRADITIONAL, BOOTSTRAP STANDARD DEVIATION, PERCENTILE, BIAS-CORRECTED, AND BIAS-CORRECTED PERCENTILE

	Traditional		Bootstrap Std. Dev.		Percentile		Bias-Corrected		Bias-Corrected Percentile	
				95% Confidence Intervals						
*MC*1	0.040	0.058	0.039	0.059	0.039	0.059	0.038	0.058	0.036	0.057
*MC*2	0.044	0.083	0.043	0.084	0.043	0.084	0.042	0.083	0.039	0.080
*MC*3	0.016	0.087	0.014	0.090	0.014	0.091	0.009	0.087	−0.001	0.083
*SC*1	0.359	0.517	0.357	0.519	0.350	0.516	0.346	0.512	0.339	0.506
*SC*2	0.208	0.391	0.205	0.394	0.210	0.396	0.201	0.396	0.182	0.379
*SC*3	0.038	0.200	0.032	0.205	0.031	0.206	0.020	0.200	0.003	0.188
SCE	0.754	0.958	0.745	0.967	0.745	0.965	0.739	0.962	0.699	0.945
SCOPE	0.528	0.791	0.515	0.803	0.523	0.807	0.526	0.820	0.542	0.848
*AES*11	−0.261	0.099	−0.141	−0.021	−0.146	−0.024	−0.143	−0.023	−0.143	−0.023
*AES*22	−1.446	−0.994	−1.840	−0.600	−1.870	−0.648	−1.835	−0.615	−1.823	−0.593
*AES*33	−12.440	−11.140	−13.020	−10.550	−13.040	−10.530	−13.040	−10.530	−13.030	−10.530
*AES*12	0.095	0.214	−0.030	0.339	−0.017	0.353	−0.018	0.349	−0.020	0.348
*AES*13	−0.112	1.094	0.266	0.716	0.263	0.717	0.260	0.711	0.260	0.714
*AES*23	0.667	3.179	1.264	2.581	1.270	2.558	1.288	2.575	1.279	2.561
				90% Confidence Intervals						
*MC*1	0.041	0.057	0.041	0.057	0.041	0.057	0.039	0.056	0.038	0.056
*MC*2	0.047	0.080	0.046	0.080	0.046	0.080	0.046	0.080	0.042	0.078
*MC*3	0.022	0.082	0.020	0.084	0.021	0.084	0.016	0.081	0.009	0.077
*SC*1	0.372	0.504	0.370	0.506	0.369	0.503	0.366	0.501	0.356	0.496
*SC*2	0.223	0.376	0.220	0.378	0.220	0.378	0.218	0.378	0.202	0.367
*SC*3	0.051	0.187	0.046	0.191	0.049	0.189	0.039	0.186	0.031	0.180
SCE	0.770	0.942	0.763	0.949	0.766	0.950	0.761	0.944	0.739	0.936
SCOPE	0.549	0.770	0.538	0.780	0.542	0.781	0.551	0.788	0.558	0.807
*AES*11	−0.232	0.070	−0.132	−0.030	−0.133	−0.032	−0.132	−0.030	−0.132	−0.030
*AES*22	−1.410	−1.030	−1.740	−0.699	−1.765	−0.732	−1.731	−0.706	−1.728	−0.688
*AES*33	−12.330	−11.240	−12.820	−10.750	−12.850	−10.770	−12.830	−10.760	−12.820	−10.760
*AES*12	0.104	0.204	0.000	0.309	0.006	0.309	0.005	0.305	0.005	0.305
*AES*13	−0.015	0.997	0.302	0.680	0.298	0.677	0.292	0.674	0.295	0.675
*AES*23	0.869	2.977	1.370	2.476	1.362	2.475	1.375	2.493	1.374	2.489

lem of bias does not affect the input measures (the *AES*'s) differently from the output measures, while skewness is more of a problem for the output measures than for the input measures.

Table 3 gives the 95% and 90% confidence intervals using the five alternatives. Most of the changes from the traditional confidence intervals come from using the bootstrap standard deviation as an estimate of the standard error. For the output measures, small differences between the traditional and bootstrap standard error estimates result in similar confidence intervals. For the *AES*s large differences between the traditional and bootstrap standard error estimates cause corresponding differences in the confidence intervals. Similarity of the other four alternative bootstrap confidence intervals is evidence that bias and skewness are unimportant in our application.

The bootstrap alternatives resolve two ambiguities that exist if the traditional confidence intervals are used. Looking at the traditional confidence intervals, we cannot conclude that AES_{11} is negative or that AES_{13} is positive, even at the 10% significance level.

TABLE 4.—PERCENTAGES OF BOOTSTRAP PARAMETER SETS THAT SATISFY CONCAVITY AND ALL REGULARITY CONDITIONS AT THE MEANS OF THE SAMPLE DATA AS THE NUMBER OF BOOTSTRAP RUNS (B) INCREASES

	$B = 100$	$B = 250$	$B = 500$	$B = 1000$
Concavity	100%	98.9%	98.8%	98.9%
All Regularity Conditions	99%	98.4%	98.4%	98.2%

Negativity of AES_{11} is a regularity condition implied by cost minimization. All the bootstrap confidence intervals resolve this ambiguity at the 5% significance level. On the other hand, the bootstrap alternatives introduce another ambiguity at the 5% level on the sign of MC_3, which is another regularity condition.

Our empirical results indicate that bootstrap resampling provides an alternative to traditional analysis and that this alternative yields different confidence intervals. Another benefit is to expand the scope of analysis. We do this by using the bootstrap results to establish some statistical confidence in our analysis of the cost function's regularity.

Some regularity conditions, such as symmetry and homogeneity in input prices, can be easily imposed. Others, however, cannot be parsimoniously imposed, and instead must be verified. Many researchers do not report on regularity. Those who do often consider only the means of the data. Even when regularity is evaluated at every data point, it is rare that any statistical significance is assigned to satisfaction of regularity. Statistical tests of significance are possible for marginal costs and input demands, but statistical tests for concavity are very difficult or impossible to calculate using traditional analysis. Furthermore, regularity requires the simultaneous satisfaction of these conditions, so testing them individually is questionable.

Bootstrap resampling allows us to make a statement of the statistical significance of regularity at all data points and at any point outside the sample. This extra information is particularly useful if policy is based on function estimates evaluated away from the point of approximation—for example, evaluating scope economies. In table 4, we report the frequency with which concavity and the set of all regularity conditions are satisfied when the bootstrap estimates are used. We expect this percentage to converge as the number of bootstrap estimates (B) increases. We can say, conservatively, that we are confident at the 5% significance level that the true cost function satisfies all regularity conditions at the means of our data. This indicates our estimated base model cost function is well behaved, allowing us to infer information about technology from our estimates.

V. Conclusions

Benefits of bootstrap resampling are to establish statistical confidence in the analysis of cost function regularity and to increase the accuracy of confidence intervals. The latter results from improving the standard error estimates and allowing for non-normality by adjusting for bias and skewness. In our application, improving the standard error estimate is the major gain. Improvement is greatest for the relatively complex measures of *AES*'s and *SCOPE*. This finding suggests that the first-order variance approximations may be inaccurate for complex statistics. Phillips and Park (1988) raise essentially the same point in the context of a Wald statistic and argue the need for higher order terms to be obtained by Edgeworth expansions; however, the bootstrap resampling technique is an easier remedy.

Another benefit of bootstrap resampling is avoiding the derivation of gradient vectors used in the approximate variance formula. This effort is greater when the number of parameters in the model is large (as is typically the case with "flexible forms"), and when the statistics of interest are relatively complex combinations of the model's parameters.

Bootstrap resampling substitutes computing resources for some labor. The effort required to estimate the base model is not a net cost because the base model must be estimated for both the bootstrap and traditional approaches. The only additional labor involved is writing the Monte Carlo estimation program. Computing costs of bootstrap resampling are higher than for the traditional method. However, once the bootstrap estimates are obtained, the cost of increasing the number of statistics of interest is near zero. Thus, one should be more likely to use bootstrap resampling as (1) there are suspicions of non-normality of the statistics of interest, (2) it is believed that the traditional first-order variance approximations are imprecise, (3) the number of estimated parameters increases, (4) the number of statistics of interest increases, and (5) the opportunity cost of own-time relative to computing increases.

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